

# THE DEVELOPMENT OF VIBRATION TEST SPECIFICATIONS FOR SPACECRAFT APPLICATIONS

by G. H. Klein and A. G. Piersol

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\*Consultant to Contractor.

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#### ABSTRACT

This document discusses the problem of developing vibration test specifications for flight vehicles from a broad engineering viewpoint. The specific steps related to the development of specifications are outlined, and the various procedures currently employed to accomplish each step are reviewed. The shortcomings of current procedures are then summarized with emphasis on the special problems posed by spacecraft applications. Finally, a logical implementation of state-of-the-art procedures to create efficient vibration test specifications for spacecraft is suggested and outlined. The problems associated with the suggested approach are discussed and areas in need of further study are noted.

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#### 1. INTRODUCTION

The most important single reason for the collection and analysis of flight vehicle vibration data is the need for information to guide the development of vibration test specifications. Yet, even with all the interest and attention which has been devoted to this problem, the procedures currently employed to establish vibration test specifications are often inadequate from the technical viewpoint. Because of the lack of rational and consistent quantitative procedures, the development of vibration test specifications is usually influenced more by personal judgments and the precedance of prior specifications than by an orderly scientific evaluation of available information.

The purpose of this report is basically twofold. The first purpose is to review the better known past and present procedures for developing vibration test specifications, and to summarize their shortcomings. For generality and completeness, the review covers applications for all types of flight vehicles including aircraft, although spacecraft applications are of specific interest. The second purpose is to suggest and outline a general approach to the development of vibration test specifications which will reduce the shortcomings of previous procedures. It should be emphasized that the intent here is only to outline an orderly implementation of state-of-the-art techniques, and not to propose a radically new approach to the problem.

The source material for this report includes published technical papers, government and industrial reports, and personal meetings with personnel of various aerospace companies and government agencies throughout the country. For the reader's convenience, the references for this report (presented in Section 6) are each followed by a brief description of material covered by that particular reference. Numerous additional documents and reports, other than those listed in Section 6, were reviewed during the study leading to this report. However, only those documents which contribute directly to the discussions herein are listed as references.

#### 2. PRESENT PROCEDURES FOR DEVELOPING TEST SPECIFICATIONS

There are many different detailed procedures which are currently used to create vibration test specifications. However, all these procedures include certain common general steps. These general steps are illustrated in Figure 1. The first step involves the original collection of actual environmental data and the reduction of this data into a usable form. If the flight vehicle of interest is not available, the environment must be predicted. The dotted line from data acquisition and reduction to environmental prediction means that predictions of vibration environments in new flight vehicles are often based upon actual data measured in similar past vehicles. After the environment is estimated either by direct measurement or prediction, the next general step consists of dividing the data into groups, where each group defines a local structural area or zone which will be covered by a single specified test. The grouping of the data is followed by the actual writing of a test specification. The last step is the performance of a vibration test in accordance with that specification. The dotted line from laboratory testing to specification writing indicates that the specification is sometimes influenced by the type of laboratory equipment which is available for testing.

#### 2.1 DATA ACQUISITION AND REDUCTION

#### 2.1.1 Data Acquisition

Ideally, the acquisition of flight vibration data should be based upon a carefully designed experimental plan which will assure a proper definition of the vibration environment with a known level of uncertainty. Unfortunately, such formal data acquisition plans are rarely executed in practice. The principal reason is simply the difficulty in acquiring sufficient data.

For the case of aircraft, it is often possible to collect enough data to permit the preparation of accurate vibration test specifications. Aircraft are relatively easy to instrument, and aircraft flight tests are comparatively inexpensive to perform. For the case of spacecraft, however, the data data for past flight vehicles may be used to predict environments for future flight vehicles

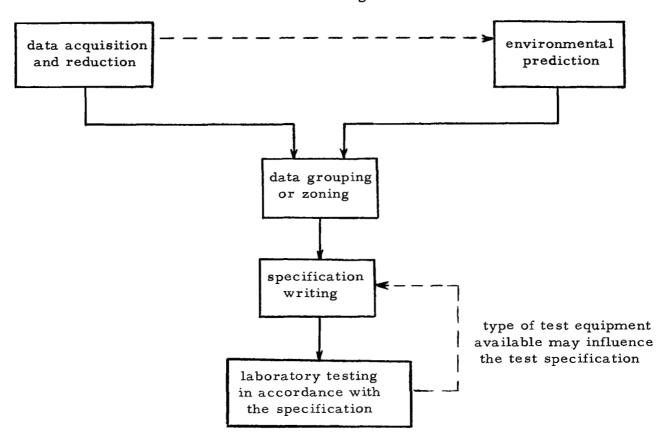


Figure 1. Basic Steps in Generating Vibration Test Specifications

acquisition problem is far more severe for two reasons. First, a significant portion of the vibration environment in spacecraft is due to such factors as aerodynamic boundary layer turbulence, maneuvering loads, staging shocks, etc., which obviously cannot be simulated by ground static firings. Second, there are practical difficulties involved in transmitting data from transducers located in a spacecraft for either launch phase or re-entry phase vibration measurements. These practical problems tend to minimize the number of vibration measurements that are available for spacecraft missions.

The problem of data acquisition for launch vehicles and missiles falls somewhere in the middle. It certainly is not as easy or inexpensive to obtain launch vehicle or missile vibration data as it is to obtain aircraft vibration data. On the other hand, launch vehicle and missile data is usually not as difficult to obtain as spacecraft data. This is true because that segment of the vibration environment produced by the acoustic excitation of exhaust gas turbulence during lift-off is more pronounced for launch vehicles and missiles, at least in lower structural regions, than for spacecraft. This segment of the environment is reasonably well simulated by ground static firings. Hence, a great deal of meaningful vibration data for launch vehicles and missiles can be acquired during ground static firing tests, which are much easier and cheaper to instrument than actual launches.

#### 2.1.2 Data Reduction

Prior to World War II when flight vehicles were principally reciprocating engine driven propeller type aircraft, flight vehicle vibration data was basically periodic or almost-periodic in nature. There were, of course, some stochastic forces inducing vibration in these aircraft, such as aerodynamic boundary layer turbulence. However, the random type vibrations were usually incidental compared to the periodic vibrations induced by the propeller blade rotation and engine firing sequence. With the introduction

of rocket and jet propulsion systems for flight vehicles following World War II, the basic nature of the vibration environment in flight vehicles was changed. Most of the vibration in rocket and jet powered flight vehicles is random in nature rather than periodic. To be specific, most of the vibration in such vehicles is induced by the turbulent mixing of exhaust gases from the rocket or jet engine and/or the turbulence produced by high speed aerodynamic boundary layers. Of course, other sources such as airborne rotating machinery, jet engine compressors, and certain types of self-excited oscillations may produce periodic contributions in the vibration. However, these periodic contributions are in most cases (excluding self-excited oscillations) small compared to the random vibration induced by stochastic forces.

The techniques required to reduce and analyze random vibration data are substantially different from those which were appropriate for periodic vibration data. Periodic (or almost-periodic) vibrations can be described by explicit mathematical functions whose pertinent properties are easily obtained from a simple harmonic wave analysis. On the other hand, random vibrations must be described in terms of statistical averages as opposed to explicit mathematical functions. Reference 1 discusses an overall program for random vibration data reduction which is expanded upon in References 2 and 3. The general approach to data reduction discussed in these references is outlined in Figure 2.

Referring to Figure 2, it is indicated that one should verify assumptions of stationarity and randomness before proceeding with data analysis. If the vibration environment is stationary, at least over some defined flight phase, the vibration properties can be described by one set of characteristics which are time invariant, at least for that phase. Otherwise, the vibration environment must be defined as a function of time. If the vibration environment is random in nature as opposed to being periodic, different operations and instruments are required for its proper analysis. Furthermore, the length of sample records to be gathered for analysis becomes critical due

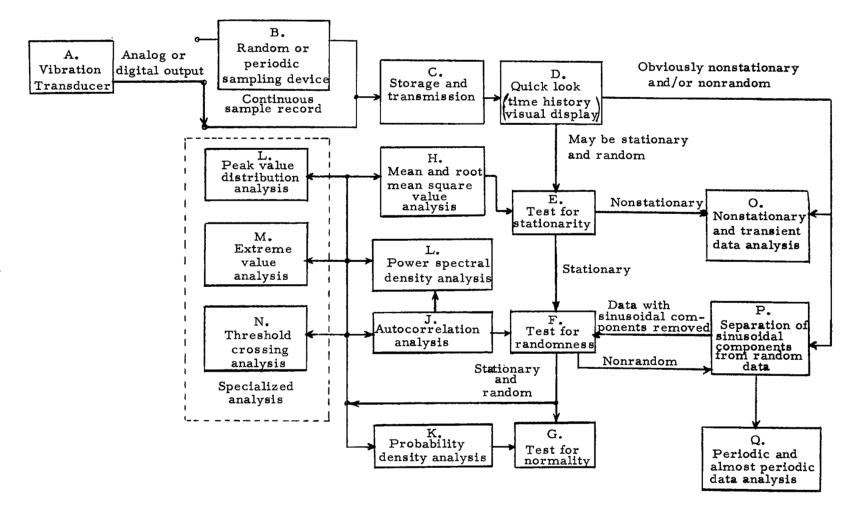


Figure 2. Procedure for Analyzing Random Vibration Data

to the inherent statistical uncertainties or sampling errors associated with random data measurements.

The verification of stationarity and randomness of vibration data does not necessarily require a formal quantitative procedure. An experienced analyst can usually detect nonstationary trends in vibration data by simple visual inspection of a time history. A lack of randomness is also discernible to an experienced analyst by visual inspection if the nonrandom component is sufficiently pronounced. On the other hand, quantitative tests are helpful for less obvious cases or when the data reduction procedure is automated. Details of quantitative procedures for detecting a lack of stationarity and randomness in sampled data are presented in Reference 2 (Section 15-17), Reference 3 (Section 2.1-2.3), and References 4 and 5.

Still referring to Figure 2, the three principal descriptive properties of random vibration data are the probability density function, correlation function and power spectral density function. The amplitude probability density function for a random vibration describes in probabilistic terms the instantaneous value relative to the rms value of the data which might be anticipated at any instant of time in the future. As for many random processes, random vibration data is often assumed to have a Gaussian (normal) probability density function. If one is prepared to make such an assumption or if the assumption is verified, the measurement of probability density functions is not required. However, it must be noted that random vibration data often deviates significantly from the ideal Gaussian form for various reasons, the most obvious of which being the nonlinear response characteristics of flight vehicle structures. Generally speaking, correlation functions yield no new information that is not available from a power spectral density function. This is true because correlation functions and power spectral density functions for stationary random data are Fourier transform pairs. Of course, in certain cases, correlation functions may present desired information in a more convenient format.

Other types of analysis in Figure 2 are sometimes employed depending upon the desired applications and specific requirements. For example, threshold crossings and peak value distributions are of considerable interest to such problems as structural fatigue damage and equipment collision predictions. Extreme value analysis is of interest to the prediction of catastrophic failures. Furthermore, there are other types of analysis which are not indicated in Figure 2. The data reduction procedures outlined in Figure 2 apply only to the problem of analyzing single sample records. Additional information is available from certain joint properties of the records, such as cross-spectral density functions and joint probability density functions.

A broad review of the instruments and techniques required for the reduction of random vibration data is presented in Reference 3, which is the basis for analysis procedures currently used by several NASA agencies and others. Note that Reference 3 outlines digital as well as analog techniques of data reduction. Generally speaking, the most important single descriptive property of random vibration data for applications to the vibration test specification problem is the power spectral density function, or some similar measure of spectral composition. Although cross-spectra measurements are required for certain advanced prediction procedures to be discussed in the next section, joint statistical measurements from two or more sample records generally yield information which is of more interest to structural research problems than to test specification problems. To a lesser degree, the same is true of probability density functions and correlation functions for single sample records.

There is a second and more practical reason why the power spectral density function is the single most important statistical property of random vibration data. The control of modern random vibration testing machines is basically a frequency domain control. The source for these machines is a random noise generator which creates a random signal with an approximately Gaussian probability density function and a relatively

uniform power spectrum over a wide frequency range. The vibration testing machine includes filtering networks which permit the power spectrum to be shaped to any desired form. It is not so easy to shape the probability density function for the signal. Hence, it follows that the principal input for a vibration test specification must be a spectral composition for the desired vibration test. In turn, since this is the most important parameter for the vibration test specification, it is the most important single parameter to be reduced from acquired flight vehicle vibration data. A detailed review of the practical measurement and interpretation of power spectra for vibration problems is presented in Reference 6.

For the special case where a vibration is stationary, at least over specific phases of a flight, the environment can be defined by one set of descriptive properties which apply to any instant of time during a stationary phase. If the vibration environment is not stationary, as is true for spacecraft, missile, and launch vehicle vibration, then the environment must be described as a function of time. This tends to complicate the data reduction procedures.

In past years, multiple filter type power spectral density analyzers have been developed which produce a continuous measurement (using short averaging times) of a frequency spectrum versus time for nonstationary random data. Attention is called to References 7,8,9, and 10 which discuss the design and use of such spectrum analyzers for the continuous reduction of nonstationary random vibration data. Unfortunately, there are some cases where time trends in nonstationary vibration data are too rapid for really effective application of short time averaging analysis techniques. An example would be the vibration environment during launch of a high acceleration surface-to-air, or air-to-air missile. For these cases, ensemble averaging data analysis techniques (averaging over a collection of records) can be applied to determine the pertinent characteristics of the environment as a function of time, if sample records are available from many (at least 10)

repeated flights. Orthogonal polynomial averaging procedures have been proposed for those cases where only a few or perhaps one sample record is available. These more advanced techniques for nonstationary data analysis are developed in Reference 2 (Sections 2-6) and References 11 and 12.

For the case of very short term nonstationary data where only one or, at most, a few fluctuations are present (transient or shock data), two additional data reduction procedures are often used. These are the Fourier spectrum and the shock spectrum for the data. The Fourier spectrum is simply the Fourier transform of the transient amplitude-time history. The shock spectrum is a plot of the response for a hypothetical single degree-of-freedom system to the transient, as a function of the natural frequency for the system. The use of these analysis techniques is developed in Reference 13.

#### 2.2 ENVIRONMENTAL PREDICTION

Often the engineer is faced with the problem of having to test components for a flight vehicle before the vehicle has been built or before actual data can be collected. In order to arrive at a reasonable test criterion, the vibration environment of the flight vehicle must be predicted. In broad terms, there are two general approaches to the vibration prediction problem. The first approach involves techniques which will be referred to as gross prediction techniques. The second approach involves techniques which will be referred to as custom prediction techniques.

#### 2.2.1 Gross Prediction Techniques

A gross prediction technique is one which does not require a specific knowledge of the structural characteristics of the vehicle of interest, or the details of the anticipated environment. Gross prediction procedures are based upon broad empirical correlations between flight conditions and vibration environments which are arrived at by averaging the vibration response characteristics measured in a wide class of flight vehicles. In most cases, the correlation is developed between an exterior sound pressure

level and a resulting vibratory acceleration response. The exterior sound pressure level is established from either boundary layer turbulence due to transonic or maximum dynamic pressure flight, or the sound pressure level due to rocket or jet engine exhaust gas mixing.

One of the earliest gross prediction techniques to be formally outlined, Reference 14, was based principally upon jet aircraft data. The approach was extended to include missile vibration data in Reference 15 (Part II), and Reference 16. Gross prediction techniques have been widely used with moderate success for vibration predictions in aircraft and long-range missiles by many aerospace companies and government agencies. However, there is increasing interest in more refined prediction techniques of the type to be discussed next.

#### 2.2.2 Custom Prediction Techniques

A custom prediction technique is one which takes into account at least some of the specific characteristics of the structure in question as well as the environmental conditions. There are three basic approaches to custom prediction as follows.

- (a) Predictions based upon measured or computed frequency response functions (or impedance functions) and excitation functions.
- (b) Predictions based upon detailed model studies.
- (c) Predictions based upon extrapolations of data from previous vehicles.

The first procedure involves a direct analytical computation of vibration responses at various points on continuous elastic structures based upon explicit expressions for the dynamic characteristics of the structures and the excitations. The application of this approach is well developed theoretically in References 17, 18, 19, and elsewhere. However, the

applications in practice to anything other than the simplest types of structures (beams and plates)have produced disappointing results to date. One difficulty has been the accurate determination of normal mode shapes for the structures in question, particularly when they are complicated shell type structures. Another difficulty has been the accurate determination of spatial correlation functions (cross-spectra as a function of distance) for the excitations. These quantities are fundamental to the direct analytical approach. A final problem is simply the excessive amount of computation required to solve the necessary equations. These difficulties are clearly illustrated and discussed in References 19 and 20.

Various simplifications of the direct analytical approach which will reduce the above difficulties are currently being studied and applied. One technique is to consider the distributed structure as a finite number of single input-output systems, and the distributed excitation as a finite number of point forces. The power spectral density function for the response at any point can then be calculated from the following equation.

$$G_{y}(f) = \sum_{i=1}^{N} \sum_{j=1}^{N} H_{i}^{*}(f)H_{j}(f)G_{ij}(f)$$

where

 $G_{v}(f)$  = power spectral density function for the vibration response

G<sub>ij</sub>(f) = cross-spectral density function between excitations at input points i and j

H<sub>j</sub>(f) = frequency response function between input point j and
the response point

H;\*(f) = complex conjugate of frequency response function between input point i and the response point.

A detailed development of this approach is available from Reference 21.

Another simplified technique is to apply modal density-energy concepts as summarized in Reference 22. This approach uses statistical ideas and concepts from room acoustics to gain an approximation for the multi-mode response of an elastic structure subjected to reverberant acoustic fields. The approach appears promising, although its usefulness has not yet been verified by practical experience.

Referring to the model study approach to prediction, mechanical scale models have been used for many years to study and predict the aeroelastic and flutter characteristics of flight vehicles. The extension of such model studies to investigate the localized vibration response characteristics of flight vehicle structures has also been attempted. Modelling laws for shock and vibrations of elastic structures are discussed in Reference 23 with special developments for spacecraft structures subjected to random excitation presented in Reference 24. Unfortunately, it can be very expensive and difficult to manufacture mechanical models which have sufficient detail to permit an accurate study of localized vibration effects. Another possible approach is the use of passive analog models, as opposed to mechanical models. Structures may be investigated either directly on a passive analog computer or on a digital computer using passive analog concepts. The derivation of passive analog circuits for three dimensional elastic structures is discussed in Reference 25.

The third of the custom prediction procedures is the most common approach used in practice. Various different formulae for predicting the vibration response in some new vehicle by extrapolating data from some previous vehicle have been developed over the years. The most common extrapolation formula used for acoustically excited structural vibrations is as follows.

$$G_n(f) = G_d(f) \frac{P_n(f)}{P_d(f)} \left(\frac{M_d}{M_n}\right)^2$$

where

G(f) = power spectral density function for the vibration response

P(f) = power spectral density function for the acoustic pressure impinging on the structure

M = structural mass per unit surface area

n = new vehicle

d = data vehicle

The above formula, in one form or another, is suggested and used in References 26, 27, 28. An additional factor is sometimes employed to account for the weight of a component which will be attached to the structure of the new vehicle, but was not present in the data vehicle. Furthermore, other formulae are used to account for vibrations induced principally by direct mechanical excitation from a rocket or jet engine. Reference 28 illustrates how some of these relationships were developed for launch vehicles.

#### 2.3 DATA GROUPING (ZONING)

The vibration environment at different points on the structure of a flight vehicle varies widely. Hence, if a vibration test specification were created to conservatively apply to all components on the vehicle, some of the components would clearly be severely overtested. It is for this reason that flight vehicles are often divided into structural areas or zones, so that a different vibration test specification can be written for the components in each of several zones. At the extreme, a vibration test specification could be created for each individual component of interest. However, this would clearly require a great deal of accurately measured data if a separate specification for each component is to be justified. Thus, the procedure of zoning a flight vehicle is basically a compromise between degree of overtesting and data volume.

Actual zoning techniques vary widely in practice. In some cases, particularly for spacecraft component test specifications, a single zone is used to cover the entire flight vehicle. This is usually done where there is not sufficient vibration data available to describe the environment with the accuracy needed to establish proper zones. In a few cases, the creation of custom specifications for every component in a flight vehicle (a zone for each component) has been attempted. In other cases, the vehicle is zoned on a regional basis, but not on a basis of structural design. In other words, the nose of a flight vehicle may be distinguished from the tail, but the vibration on basic frame structure is not distinguished from the vibration on light skin sections.

The most effective approach to zoning in current use appears to be one based on both vehicle regions and structural design. That is, not only is the nose of the flight vehicle distinguished from the tail, but the basic frame structure in the nose is distinguished from the light skin sections in the nose. Sometimes the breakdown is extended to include a dozen or more vehicle regions and perhaps three or more types of structures in each region. The zoning of the Saturn vehicle outlined in Reference 29 is a good illustration of this approach.

#### 2.4 SPECIFICATION WRITING

In general, the currently accepted conceptual approaches to writing a vibration test specification may be broadly divided into two categories as follows:

- (a) simulation of the actual environment
- (b) simulation of the damaging effects of the environment

Approach (a) leads to a test specification which presumably simulates the actual environment, at least in terms of its main characteristics. For example, if the measured or predicted vibration environment

is basically random in nature with a spectral density of G(f) g<sup>2</sup>/cps and a total duration of T seconds, then the specification would call for a random vibration test with a test level and time duration similar to those measured or predicted.

It might appear at first that the optimum specification in terms of environmental simulation would be one requiring exact reproduction of the measured environment. In other words, one could obtain actual tape recordings of the flight vehicle vibration environment at various structural locations of interest, and use these tape recordings as the input to the vibration testing machine. Unfortunately, this exact reproduction approach is not feasible for a number of practical reasons including the following.

- (a) The direct reproduction procedure would require that the vehicles and components of interest be flown prior to creating vibration test specifications. However, the purpose of the vibration test is to qualify components before they are flown in the vehicle of interest.
- (b) The procedure would require a tape recording of the vibration response at every point on the vehicle structure where a component is to be attached.
- (c) For the case of components with multiple point attachments where the vibration at each attachment point is different, there is the problem of deciding which vibration record will be used for the test.
- (d) No statistical variations can be considered. For example, there is no reason to believe that the individual flight from which measurements were obtained is necessarily representative of the most severe flight to be anticipated in the future.

The most successful approach to simulation of the actual environment is to design a similar but contrived vibration environment based upon the available sample data. The resulting test level may be based upon the maximum levels observed in the collection of measurements within any one zone so that the resulting specification will conservatively apply to all structural locations in that zone. Furthermore, the test levels may be increased by an appropriate factor to account for uncertainties in the determination of the environment. However, the basic philosophy is still that of direct environmental simulation.

The principal advantage of approach (a) is that no assumptions need be made concerning the mode and mechanism of possible failures in the structure or equipment to be tested. The principal disadvantage is clearly the problem of simulating all features of the actual environment, particularly its duration. For the case of long service life flight vehicles such as piloted aircraft, the vibration environment may have a total duration of many thousands of hours. A direct simulation test is obviously not feasible in this case. However, direct simulation is quite applicable to the case of missiles, launch vehicles and spacecraft where the vibration service life is relatively short in duration.

Approach (b) recognizes that flight vehicle vibration environments cannot always be accurately simulated in the laboratory, particularly in terms of duration. By basing the test criterion only upon a simulation of the damaging effects of the environment, "accelerated" tests can be derived where the test duration is decreased at the expense of increased test levels.

The principal advantage of approach (b) is that it permits the specification of vibration tests which simulate thousands of hours of service life with a few hours of testing. The principal disadvantage is the problem of establishing an acceptable criterion for equivalent damage.

The specific details of actual procedures for writing vibration test specifications vary widely among the different companies and government agencies. However, the more commonly used procedures do have certain pertinent features which will now be discussed. It should be emphasized that the specific procedures discussed here are not being recommended or endorsed in any way. These are simply procedures in current use.

#### 2.4.1 Environmental Simulation Procedures

The most common approach in this category is to write a test specification which exceeds the measured or predicted vibration levels at all frequencies for all data in any given zone. This technique is sometimes referred to as the envelope approach. For the case of a periodic vibration environment, all the data available for a given zone in the flight vehicle is plotted as amplitude versus frequency. An envelope is then drawn which contains all the data points. A similar approach is used for a random vibration environment where the envelope is drawn to cover all the peaks of the power spectra for all data in a given zone.

In either case, the envelope is usually fitted to consist of only two or three straight lines for ease of simulation. This resulting envelope becomes the vibration test specification. A sinusoidal vibration test is used for periodic environments and a broadband random vibration test is used for random environments. Sometimes a combination sinusoidal-random test is employed. The duration of the test is at least as long as vibration service life for the component to be tested. Hence, the procedure is most applicable to short service life vehicles such as missiles, launch vehicles and spacecraft.

In many cases, the technique used to arrive at an envelope involves more than simply covering all measured or predicted levels. First of all, it is desirable to allow for uncertainties in the measured or predicted data. Furthermore, since vibration measurements or predictions are rarely

available for all points of interest, it is desirable to allow for uncertainties in the vibration levels at points which were not measured or predicted. These uncertainties are sometimes allowed for by adding a factor which is based purely upon an educated guess of the specification writer. However, in recent years, more quantitative procedures have been introduced which involve at least rudimentary statistical considerations.

One approach is to establish test levels based upon an assumed sampling distribution for the power spectral density function of the structural vibration in a given zone. For convenience, the power spectral density function is usually reduced to mean square values in narrow contiguous frequency intervals so that it may be described by a finite number of frequency points. A sample mean value and variance is then computed for the narrow band mean square values in each frequency interval from the measured or predicted data in that zone. Based upon these sample values and the assumed sampling distribution, an upper limit for the mean square vibration in each frequency interval is estimated at any desired percentile level. For example, an upper limit which would exceed the vibration levels for 95% of the points in that zone would be estimated using the 95 percentile level of the assumed sampling distribution. The upper limits for the mean square values in the contiguous frequency intervals can then be used to define a power spectrum for the vibration test to be specified for that zone. The test can be made as conservative as desired by using higher or lower percentile levels to establish the test levels.

A number of different sampling distributions for vibration measurements have been assumed at one time or another, but the log-normal distribution has been the most widely used for data in the form of mean square values in narrow frequency intervals. An example of the above approach using a log-normal sampling assumption is presented in Reference 27.

At least one agency has approached the problem of establishing test specifications at some desired percentile level by the application of an

empirical relationship arrived at by evaluation of large quantities of past flight vehicle vibration data. This was done by NASA Marshall Space Flight Center to arrive at the test level selection procedure outlined in Reference 29. Note that the procedure in Reference 29 applies to over-all rms vibration levels rather than narrow bandwidth mean square vibration levels. The use of over-all vibration levels to establish a specification level tends to produce a less conservative test than for the case where narrow bandwidth levels are used.

For the case of random vibration environments, the direct simulation approach to specification design can place a severe burden on testing facilities. For this reason, a number of modifications to the above direct simulation procedures have been proposed over the years. In some cases, these modifications consist simply of an envelope approach where engineering judgment is used to partially discount spectral peaks believed to be unrepresentative based upon impedance considerations. In other cases, a test is specified which consists of a low level random vibration background with superimposed high level narrow bandwidth peaks. The broadband background is established by enveloping data where all spectral peaks are totally discounted. Narrow bandwidth random vibration is then used to simulate the spectral peaks in the measured or predicted environment. Still another approach is to use only a swept narrow bandwidth random vibration, as advanced by Reference 33. All of these modifications are intended to reduce the required testing machine force capability.

In some instances, a need to limit the force required of testing machines has resulted in the use of sinusoidal vibration tests to simulate random vibration environments. In this case, some criterion for equivalence between random and sinusoidal vibrations must be assumed. The most common approach is to assume that structural fatigue damage is the mode and mechanism of failure. Various specific equivalence formulae have been developed for this case, but most are simply extensions of ideas developed by

Miner in Reference 30 and specialized for random environments by Miles in Reference 31. Relationships based upon criteria other than fatigue damage have also been suggested. One of the better known is a peak criterion presented in Reference 32. It should be emphasized, however, that once a sinusoidal substitution for random vibration is made, the specification philosophy is really no longer that of environmental simulation, but of damage simulation as discussed in the next section.

#### 2.4.2 Damage Simulation Procedures

The most commonly used damage simulation procedures are based upon a fatigue damage criterion. In other words, a vibration test is established that will produce fatigue damage to the component being tested which is equivalent to the fatigue damage expected in actual service. To accomplish this end, it is assumed that fatigue is the only mechanism of failure, that some classical hypothesis for fatigue damage accumulation is valid, and that all parts which could fail are subjected to stresses which are above the endurance limit but below the elastic limit for the structural materials involved. This allows one to replace a long duration, low intensity vibration environment with a short duration, high intensity vibration test. Hence, the procedure is most applicable to long service life vehicles such as airplanes.

The basic ideas for this approach to specification design were first advanced on a rational basis in References 34 and 35, which cover work sponsored by the USAF at Wright-Patterson AFB as far back as 1953. Refinements of this early work have been made by a number of investigators, but the basic concept is the same. By assuming an S-N curve for the structure in question, the amount of fatigue damage accumulated during its expected service life can be estimated. A test which will produce the same amount of damage in a much shorter period of time is then derived. The damage accumulation criterion of Reference 30, or some modification thereof, is assumed. The damage caused by random vibration environments is estimated using the concepts established in Reference 31.

Either random or sinusoidal vibration inputs may be used for an equivalent damage test. The procedures for selecting an appropriate magnitude and duration for the random or sinusoidal test are summarized in References 36 and 37. Because this general approach to writing test specifications is well-defined and thoroughly reviewed in the literature, no more discussion will be included here.

#### 2.4.3 Combined Environmental-Damage Simulation Approach

Another approach to specification writing is based upon combining the desirable features of equivalent damage concepts with those of direct environmental simulation. A test level is arrived at using an envelope approach, as discussed in Section 2.4.1. However, the test duration is limited to that time necessary to accumulate approximately  $5 \times 10^6$  cycles of vibration. Empirical data indicates that the endurance limit for most materials used in flight vehicle components is such that a fatigue failure after this time period is not likely.

For the case of sinusoidal vibration environments, the time required to accumulate  $5 \times 10^6$  vibration cycles may be computed directly from the sinusoidal frequency. For the case of random vibration environments, the response characteristics of the component being tested must be considered to establish the time needed to accumulate the equivalent of  $5 \times 10^6$  cycles. If it is assumed that the principal vibration response of a component is occurring at the frequency of its fundamental resonance, the number of vibration cycles experienced by the test item can be considered equal to the product of resonant frequency and test time. For example, if the lowest resonant frequency of the component is 100 cps, then  $5 \times 10^6$  cycles will be realized in a 14-hour test. Hence, even if the actual exposure during service life is much longer than this time, a 14-hour test would be considered adequate. It is obvious that this approach is not practical for a test item which has a very low resonant frequency.

The main purpose of the above approach is to obtain a test for long service life vehicles which does not require the assumption of specific S-N curves for materials, as required for the procedures in Section 2.4.2., and which increases the probability of detecting failures other than those due to fatigue. See Reference 16 for an example of this approach.

#### 2.5 LABORATORY TESTING

1

The ultimate conclusion to the creation of a test specification is its implementation in the laboratory. Nearly all laboratory vibration tests performed today are accomplished by means of electrodynamic type vibration testing machines. These machines consist of a large field coil enclosing a moving armature which is constrained to rectilinear motion. The component to be tested is affixed rigidly to the vibration testing machine armature. Vibration is then delivered directly to the component to be tested by electromagnetic excitation of the armature. The armature driving signal may be delivered from either an electrical alternator or an electronic power amplifier. If an electrical alternator is used as the driving source, only a sinusoidal armature motion can be obtained. If a complex or random armature motion is required for the test, an electronic power amplifier must be employed as the driving source.

Although electrodynamic vibration testing machines are basically force generating devices, the vibration delivered by the machine during a test is usually regulated and controlled on the basis of armature motion. For the case of sinusoidal vibration tests, a simple servo-mechanism is usually employed to automatically adjust the armature signal level and produce the desired motion of the armature at various frequencies. For the case of random vibration tests, the power spectrum of the armature signal is usually shaped using a collection of contiguous narrow bandpass filters to produce the desired power spectrum for the armature motion.

It should be mentioned that laboratory dynamic testing often involves shock testing machines as well as vibration testing machines. The primary difference is that a shock testing machine delivers only one cycle of motion while a vibration testing machine delivers repeated cycles of motion. In other words, the shock testing machine delivers a transient dynamic environment as opposed to a relatively continuous dynamic environment. Modern shock testing machines are designed to permit the selection of the detailed characteristics for the single cycle of motion imparted by the machine. For example, the machine can be set to deliver a single cycle of motion resembling a half sine wave, a terminal peak sawtooth wave, a triangular wave, and other such desired waveforms. This flexibility permits a wide range of freedom in the design of shock test specifications.

#### 3. MAJOR SHORTCOMINGS OF PRESENT PROCEDURES

The major shortcomings posed by the procedures for arriving at specifications, as reviewed in Section 2, will now be discussed. Specific deficiencies for spacecraft applications are emphasized.

#### 3.1 DATA ACQUISITION AND REDUCTION

In the area of vibration data acquisition, the principal deficiency is simply the ever present problem of obtaining a sufficient amount of data. This problem can sometimes be reduced by carefully planned flight test experiments and optimum data gathering procedures. Far too often, insufficient attention is given to the selection of transducers and their location, the length and number of measurements, the methods of recording, and other such vital matters.

In the area of vibration data reduction and evaluation, the general procedures presented in Reference 3 and the specific procedures for power spectral density analysis detailed in Reference 6 are reasonably thorough and complete. However, the procedures in these references do have one important deficiency. Their application to the reduction of nonstationary vibration data is not clear. As mentioned in Section 2.1.2, the use of short averaging time spectrum analyzers for continuous analysis of nonstationary data, as covered in References 7, 8, 9, and 10, is widespread. However, the statistical accuracy of the continuous spectra produced by such instruments is usually poor, These matters are currently being studied in more detail, as indicated by Reference 38.

For those cases where the vibration data is changing very rapidly with time, the problem is more difficult. Theoretically, the technique of References 11 and 12 are applicable for analyzing such data. However, there is some question as to how the analysis should be interpreted to create a test spectification. If the environment is changing rapidly relative to the response time of the component to be tested, the environment should perhaps be thought

of as a shock and not a vibration. For this case, a shock test based upon a shock spectrum measurement might be the best approach. If the environment is changing slowly relative to the response time of the component to be tested, then the exact time varying characteristics of the environment are not so critical, and a series of short stationary vibration tests can be derived which will provide an adequate simulation. Of course, the decision as to whether or not the environment is changing rapidly with respect to the response time of the component is not always clear. Recent theoretical and computer studies of this problem are presented in Reference 39.

#### 3.2 ENVIRONMENTAL PREDICTION

The survey of vibration prediction techniques in Section 2.2 reveals many deficiencies. Of course, criticism is very easy here since the prediction problem is one of the most complex and difficult areas associated with the generation of vibration test specifications. However, it does appear that certain phases of the prediction problem could be improved without a major advance in the state-of-the-art.

First, for extrapolation type prediction methods, most extrapolation formulae presently used are based primarily upon the surface mass density of the structure. Surface density is indeed the critical parameter for vibrations at those frequencies where the structural response is "mass controlled" (frequencies well above primary resonances). Furthermore, the vibration of flight vehicle structures at higher frequencies can perhaps be considered as mass controlled with reasonable accuracy. It is clear, however, that such extrapolation formulae are useless for predicting lower frequency vibration which is strongly influenced by structural stiffness and damping characteristics. It appears that more attention should be given to the possibility of extrapolations which consider these additional factors.

Second, for prediction methods which require an estimate for the sources of vibration excitation, the usual approach is to limit attention to

the acoustic inputs from jet or rocket exhaust gas turbulence and aerodynamic boundary layer turbulence. It is true that these two inputs are the predominant sources of excitation in many cases. However, there are situations where other sources of vibration may be significant. For example, principal sources of flight vibration for a spacecraft might include the fundamental bending response of the launch vehicle to control system loads and the direct structure-borne vibration from the rocket motor. In some cases, on board equipment such as high speed rotating machinery will produce vibrations which are more significant in local areas than the general vibration background due to the pressure fluctuations generated by exhaust gas mixing and/or boundary layer turbulence.

Finally, the vibration environment resulting from ground transportation and handling of flight vehicle components may be more severe from the viewpoint of structural damage than the future flight vibration environment. This is particularly true for the case of spacecraft where the transportation and handling environment could extend over several hours while the total flight environment involves only a few minutes of significant vibration exposure.

#### 3.3 DATA GROUPING

The proper zoning of a flight vehicle is an important key to accurate test specifications. At the present time, the zoning procedure is accomplished in a relatively arbitrary manner. At best, zones are selected on a basis of regional location and structural design. This approach does not necessarily minimize the variation of vibration levels within each of a fixed number of zones, which is really the ultimate goal of zoning. For example, vibration of primary structures near the tail of an airplane may be similar in intensity and spectral characteristics to vibration of secondary structures near the nose. However, a zoning procedure based on either regional location or structural design would place these two measurements into different groups.

It is clear that a zoning procedure based upon some sort of data equivalence criterion would be a more efficient way to approach the problem.

#### 3.4 SPECIFICATION WRITING

The principal deficiencies associated with the writing of test specifications are related to the assumptions employed to derive test levels. These specific assumptions are noted for the various writing procedures outlined in Section 2.4. For example, the damage simulation approach discussed in Section 2.4.2 assumes that the principal mode of failure is fatigue damage in accordance with some specific damage rule. On the other hand, the combined environmental-damage simulation approach discussed in Section 2.4.3 assumes that all materials have an endurance limit which is reached in less than  $5 \times 10^6$  cycles.

Besides the above mentioned specific assumptions, there are many general assumptions which apply to the various procedures, although they are not specifically noted. The most important of these general assumptions which produce deficiencies in the resulting specifications are listed below.

- 1. Mechanical impedance considerations are often ignored.
- 2. Various types of nonlinearities are ignored.
- Continuous stationary vibration tests are often specified to represent highly nonstationary vibration environments.
- 4. There is no clear statistical basis for the specification.

These various specific and general deficiencies in vibration test specification writing procedures will now be discussed.

#### 3.4.1 Mode and Mechanism of Failure

As noted in Section 2.4, many test specification writing procedures in current use are based upon the assumption that fatigue is the only mode of failure. This assumption may be reasonably valid for the case of components which consist solely of inactive structures. However, the assumption that

fatigue damage is the sole mode of failure may be unrealistic for various types of assembled operational components. In order to at least qualitatively evaluate the limitations of this assumption, a list of other types of failures which might occur is given below. This list is taken from Reference 15, which expands upon this subject.

- 1. Direct mechanical vibration or acoustic excitation of electronic vacuum tubes can produce oscillations in tube elements such as filaments, grids, cathodes, and plates. The relative positions of these elements may be critical and fluctuations of this nature may temporarily change the tube characteristics, increase the electrical background noise in the tube, or perhaps cause arcing or shorting of the elements. Conventional tube elements are often long and slender, and are generally cantilevered from the base with natural frequencies ranging from 500 cps to 8000 cps. Hence, vacuum tubes tend to be very susceptible to malfunction when subjected to dynamic excitations in this frequency range.
- 2. Acoustic excitation produces nearly uniform compression of small components. The result is that a capacitor may act as a microphone creating electrical noise in the circuit with little or no permanent damage to the capacitor. Furthermore, large chassis are often excited acoustically to cause intense vibration of attached elements.
- 3. Relay chatter is a frequent type of failure in which the contacts of an open relay oscillate and accidentally close an open circuit or open a closed circuit. Oscillation of the contacts of a closed relay may change the normal pressure between the contacts so that the contact area fluctuates, changing the electrical resistance of the contacts and the current flowing through the relay. Furthermore, the relay may eventually freeze in an open or closed position.

- 4. Equipment components such as resistors and condensers are often located adjacent to large flat surface panels such as the chassis.

  Either direct mechanical or acoustic excitation of the flat panel may cause impacts with the components and produce breakages. This is especially true of carbon resistors.
- 5. The wires connecting equipment components such as resistors and condensers often experience stretching due to the vibration induced distortions of the chassis. The same effect occurs when the components resonate on their connecting wires. The resulting high bending moments at the terminal posts along with the accumulation of fatigue damage in the wires will often produce failure in a relatively few number of cycles. Wire breakage is the most common cause of failure in electrical and electronic equipment.
- 6. Vibration often causes equipment wiring to rub against neighboring components so that the insulation on the wires wears away, producing short circuits. This is also a common failure of multiconductor cables.
- 7. The vibration induced bending of coaxial cables can often produce noise voltages which cause temporary circuit malfunctions.
- 8. Dynamic loads on rotating equipment can produce galling of bearings and bearing races which in turn may produce serious mechanical failures.
- 9. Equipment involving optical systems may drift out of alignment and malfunction due to continued exposure to vibration.

# 3.4.2 Linearity of Damage Accumulation

The hypothesis that a fatigue failure is due to the irreversible accumulation of damage caused by repeated stress cycles is generally accepted as an appropriate model for metal fatigue. However, the hypothesis may be applied in many different ways. The most common application of the cumulative damage hypothesis is one that assumes linearity.

To be more specific, assume a structure is subjected to a repetitive load producing a maximum stress level of  $S_1$ . Further assume that the number of such cycles required to produce a fatigue failure is  $N_1$ . Then, if the structure is subjected to  $n_1 < N_1$  such stress cycles, the linear damage hypothesis would say that a fraction of the total fatigue life for the structure equal to  $n_1/N_1$  is consumed or used up. If the structure is then subjected to a repetitive stress  $S_2$  for  $n_2 < N_2$  cycles, where  $N_2$  cycles would produce failure, an additional fraction of the total fatigue life equal to  $n_2/N_2$  is consumed. Damage is accumulated in this manner until failure occurs. The linearity assumption implies that the order of application for loads with different magnitudes does not influence the total number of cycles to failure.

Experimental data discussed in Section 9.4 of Reference 1 indicate the linearity assumption for damage accumulation may produce considerable error in fatigue predictions. The validity of the linearity assumption clearly influences the accuracy with which a high intensity, short duration vibration test can be used to simulate the fatigue damage caused by a varying low intensity, long duration vibration environment. There are cumulative damage hypotheses which assume nonlinear characteristics in metal fatigue damage accumulation based upon laboratory tests. However, most vibration test specifications in current use employ only the simple linear model to arrive at an equivalent damage criterion.

#### 3.4.3 Random-Sine Equivalence

Some approaches to vibration specification testing call for sinusoidal vibration alone or in conjunction with random vibration to simulate basically random environments, as discussed in Sections 2.4.1 and 2.4.2. Furthermore, due to the high cost of random vibration testing equipment and certain practical problems associated with the use of this equipment, many testing laboratories are not equipped with facilities to perform random vibration tests. Hence, sinusoidal tests are often used to simulate random vibration environments for purely economic reasons. In such situations, the following question arises. What should be the level of the sinusoidal vibration to properly simulate a random vibration?

A number of ideas have been proposed over the years to establish an equivalence between a sinusoidal and random vibration environment. A good review of these ideas is presented in Reference 15, Part III. Most of the theoretical approaches are based upon a criterion of equivalent fatigue damage. Hence, the application of such equivalence expressions involves all of the deficiencies and problems that have been discussed in Sections 3.4.1 and 3.4.2. The random-sine equivalence involves one other serious deficiency. Since a sine wave vibration can occur at only one frequency at any one time, as opposed to a random vibration which produces excitations at all frequencies simultaneously, a superposition of damage accumulation must be assumed. In other words, the accumulation of damage when each resonance is excited individually must be considered equivalent to the total damage that occurs when all resonances are excited simultaneously. superposition assumption may be highly questionable, particularly if the response characteristics of the component being tested are significantly nonlinear.

The more recent work on random-sine equivalences has been concerned principally with empirical correlations for specific types of hardware. This approach appears to be more promising than the development of general relationships with broad applications, as previously pursued.

#### 3.4.4 Mechanical Impedance Considerations

All currently used vibration test specifications establish the test levels by specifying the vibration level as a function of frequency. For the case of sine wave tests, the specified amplitude parameter may be either displacement, velocity, or acceleration. For the case of random vibration tests, the specified parameter is usually acceleration density. (The test power spectral density level is specified in g<sup>2</sup>/cps.)

If the vibration data used to write a test specification is based upon actual measurements or accurate predictions at structural locations of interest with all components mounted as in service, no problems arise. Furthermore, even if the vibration data is based upon measurements or predictions without components mounted as in actual service, the use of a motion parameter for environmental specification may still be satisfactory if the mechanical impedance of the structure is large compared to the components to be supported in service. In other words, if the mounted components do not significantly load their supporting structure, it is not necessary for the components to be installed when structural vibration measurements are obtained.

On the other hand, if the mechanical impedance of the supporting structure is not large compared to the mounted components, then the vibration response characteristics of the unloaded structure will be quite different from the vibration in actual service with all components installed. In such cases, when a vibration test specification is written on a basis of the vibratory motion of the unloaded supporting structure, the end result is a tendency to produce an overly severe vibration test. The same effect occurs when a vibration test specification is established by enveloping peaks in a measured response power spectrum. These points are discussed further in Reference 40.

Mechanical impedance simulation in the laboratory, in the sense of allowing the vibration testing machine to react in a manner similar to the actual supporting structure for the test item, is not practiced today. Therefore, the entire burden for properly interpreting the measured or predicted vibration data so that a realistic vibration specification can be designed is placed upon the specification writer. For the case of relatively large components, mechanical impedance factors should receive at least qualitative consideration in the writing of vibration test procedures. One approach is to apply a simple mass law correction to the measured or predicted vibration data. This technique is discussed and illustrated in Reference 28. More extensive discussions of mechanical impedance and its importance to the shock and vibration testing problem are presented in Reference 41.

## 3.4.5 Nonlinearities

All real structures will display nonlinear response characteristics to vibration excitation if the level of vibration is sufficiently intense. Both nonlinear stiffness characteristics and nonlinear damping characteristics are involved. In many cases these nonlinear conditions may not be sufficient to justify concern. However, there are other cases where nonlinearities may produce serious problems.

Consider the case where an accelerated vibration test is to be performed on a component by increasing the vibration test level based on an equivalent fatigue damage criterion, or for that matter any criterion. It is obvious that nonlinear response characteristics for the component will bias the desired equivalence when the vibration test level is increased. The result could be a test which is either more severe or less severe than anticipated, depending upon numerous factors.

The subject of nonlinearities and their importance to general engineering problems are widely discussed in the literature. No additional attention is warranted here. However, it should be remembered that structural linearity is indirectly assumed during many phases of various specification derivation procedures.

# 3.4.6 Stationary Testing Considerations

As noted in Section 3.1, the reduction and analysis of vibration data for high acceleration missiles can be difficult due to the rapidly changing nature of the environment. However, even if such nonstationary data is properly analyzed, there is still the additional problem of deriving an appropriate vibration test specification. Time varying vibration tests could perhaps be specified, but such testing is not commonly performed at present.

The current procedure is to use relatively short duration stationary vibration tests to simulate the environment of missiles and spacecraft. Several such short tests with different levels and power spectra might be employed to represent different pertinent conditions such as launch, transonic flight, and maximum dynamic pressure. However, a test with a continually varying power spectrum is not presently used. This tends to add some uncertainty to the true meaning of the test results. It might be more effective to simulate highly nonstationary vibration environments by a single pulse shock test rather than a continuous vibration test. This matter is in need of further study.

## 3.4.7 Statistical Considerations for Specification Design

The lack of a proper appreciation for the statistical aspects of the test specification problem is probably the most severe deficiency in present procedures. However, with the increasing interest in general reliability concepts, the importance of statistics as an everyday tool for all fields of engineering is gradually being accepted. For the case of generating vibration test specifications, there are numerous statistical uncertainties which arise in each step of the procedure. These uncertainties must be considered to arrive at a test level which will have a known probability of being as damaging as the actual environment. It should be noted that these statistical uncertainties are always in addition to the normal instrument errors that are present in the measurement, data reduction and laboratory equipments.

The more important sources of statistical uncertainties are as follows:

- 1. The sampled vibration data gathered for analysis represents the vibration response at only specific points on the structure of the flight vehicle. It is very rare that one is fortunate enough to obtain vibration data at every point of interest. Hence, there is an uncertainty associated with the use of this measured vibration data to predict the vibration environment in the flight vehicle at other points of interest which were not measured.
- 2. Sampled vibration data gathered for analysis represents the vibration environment in a flight vehicle over specific intervals of time in the past. Hence, there is some uncertainty associated with the use of this data to predict the vibration environment to be expected over all times in the future.
- 3. It is often not possible to obtain sampled vibration data from the actual flight vehicle of interest. Data from other vehicles must be employed along with theoretical considerations to predict the vibration levels in the vehicles of interest. Hence, there is an uncertainty as to how well the predicted environment represents the vibration environment in the actual vehicle of interest.
- 4. Vibration test specifications are rarely designed for each structural point of interest in the flight vehicle. The general procedure is to pool together data to establish one general specification which is applicable to a zone representing a wide range of structural locations. Hence, there is an uncertainty associated with how well this resulting specification actually represents the vibration environment in the flight vehicle for the various structural locations which are zoned together for the specification.

- 5. The actual component made available for testing is only a sample of the hardware of that design which will be produced for use in service. There is probably some variation in the fragility level (sensitivity to failure) for the production items. Hence, there is an uncertainty as to how well the fragility level of the actual test item represents the fragility level of subsequent production items.
- 6. As previously mentioned, many assumptions are often made to arrive at a test specification. For example, the specification may be based on an equivalent fatigue damage criterion where the damage accumulation is assumed to be linear, a random-sine equivalence might be employed in the test, mechanical impedance matching problems might be ignored, etc. Each of these assumptions introduces an additional uncertainty as to how well the resulting vibration test represents the desired test.

As noted in Section 2.1, it is often difficult to obtain all of the sampled data that one would normally desire for the design of a statistically sound vibration test specification. For such cases, it may be difficult to define meaningful uncertainties to guide the derivation of the vibration test. However, there is a possibility that quantitative estimates for an optimum vibration test can still be obtained by the combined application of statistical decision theory, subjective probability concepts, and good engineering judgment. Such an approach to the problem is suggested in Reference 42. It is unfortunate that more work has not been done to apply statistical decision theory to the problem of selecting optimum test levels.

#### 3.5 LABORATORY TESTING

Most of the deficiencies faced in laboratory vibration testing are the result of purely practical problems associated with the design of large vibration simulation equipment. As long as the component to be tested is relatively small and has clearly defined attachment points, few problems are involved in reproducing the vibration levels requested by the test specification. However, as the component becomes large and bulky, or its attachment points become complicated and awkward, numerous practical problems arise which make it very difficult to deliver the specified vibration environment to the component.

For the case of sinusoidal vibration tests, large components with non-linear characteristics often cause severe distortions in the applied vibration as discussed in Reference 43. For the case of random vibration tests, the problem of equalization of the vibration testing machine (shaping the proper power spectrum for the test) becomes more severe as the test item becomes larger.

As noted in Section 3.4.4, current laboratory vibration testing equipment does not incorporate provisions for mechanical impedance simulation. It is not being suggested here that this capability is necessary at the present time. There are still serious problems which would limit the effective and proper application of simulated impedance testing, even if the capability were available in laboratory testing equipment. However, future advances in the measurement and prediction of structural impedance characteristics for flight vehicles might make such a capability highly useful.

Another possible advance in testing equipment which might be useful in the near future would be provisions for performing nonstationary vibration tests. Further study of this approach is needed.

# 4. A SUGGESTED APPROACH TO THE DEVELOPMENT OF VIBRATION TEST SPECIFICATIONS FOR SPACECRAFT APPLICATIONS

Past and present procedures employed to develop vibration test specifications, and the major shortcomings associated with these procedures, have been reviewed in Sections 2 and 3. Based upon that review, an over-all approach to the development of test specifications is now suggested for the specific case of spacecraft applications.

The suggested approach evolves from a logical implementation of the state-of-the-art techniques for environmental measurement, prediction, and testing. The basic purpose is to minimize the various shortcomings associated with previous specification procedures, as discussed in Section 3. However, the approach still does pose some practical difficulties which are discussed in later sections.

#### 4.1 PHILOSOPHY OF SUGGESTED APPROACH

There are two fundamental requirements for a "good" vibration test specification which may be summarized as follows:

- (a) If a component functions properly during the specified vibration test, there should be a high probability that the component will function properly in the service environment.
- (b) If a component malfunctions during the specified vibration test, there should be a high probability that the component will malfunction in the service environment.

The first requirement means, in effect, that the specified vibration test should be at least as severe as the vibration environment to which the component will be exposed in service. The second requirement means that the specified vibration test should not be unreasonably more severe than the vibration environment to which the component will be exposed in service.

Failure to comply with the first requirement will result in undertesting, while failure to comply with the second requirement will cause overtesting.

Generally speaking, the vibration test specifications which have been created over the years have complied with the first requirement for a good specification. In other words, past and present vibration test specifications have tended to be conservative. This is true because any uncertainty as to whether or not a specification is sufficiently severe has usually been dealt with by arbitrarily increasing the specified test levels and/or durations until such uncertainty is minimized to the satisfaction of all concerned. Although this procedure will usually satisfy the first requirement for a good test specification, it obviously will tend to violate the second requirement. Hence, it has generally been in this second requirement area where vibration test specifications have left much to be desired.

To meet both requirements for a "good" vibration test specification, it is necessary to define the uncertainties associated with each step involved in developing the specification, and to reduce these uncertainties to an acceptable level. In particular, uncertainties due to assumptions should be eliminated wherever possible. This philosophy immediately suggests that the conceptual approach to writing a test specification should be one of simulating the actual environment, as opposed to simulating some hypothesized damaging effects of the environment, since fewer assumptions are required.

#### 4. 2 OUTLINE OF SUGGESTED APPROACH

An outline of the suggested approach is presented below. Each step is discussed in the next section. Although the suggested approach is intended for spacecraft applications, it is also directly applicable to the development of specifications for launch vehicles and military missiles, or for that matter, to any jet or rocket powered flight vehicle with a relatively short vibration service life. Furthermore, all steps except the last two (Steps 9 and 10) are applicable to the development of test specifications for any type of flight vehicle, including airplanes.

# Step 1: Establish the assembly level at which testing is to be performed.

This may include any one or more of the following.

- (a) individual parts
- (b) equipment packages
- (c) primary structural subassemblies
- (d) the entire spacecraft assembly

# Step 2: Establish the vibration life history that is to be covered by the specification.

This may include any one or more of the following.

- (a) factory handling
- (b) transportation environments(by truck, railroad, ship, or aircraft)
- (c) storage (handling either manually or by power equipment)
- (d) final installation
- (e) actual launch or flight environment

# Step 3: Establish the purpose of the specification.

This may include any one or more of the following.

- (a) design information tests (to obtain information for improving the design)
- (b) design evaluation tests (to evaluate the final design)
- (c) qualification tests (to formally demonstrate the design)
- (d) acceptance tests (to demonstrate that the initial quality has been retained throughout production)

# Step 4: Establish the maximum acceptable uncertainty for the specification.

The over-all uncertainty for a vibration test specification is a function of the separate variances associated with the principal steps required to write and implement the specification. Since each step generally involves independent considerations, the variances for the individual steps can be summed to arrive at an over-all variance for the predicted levels. The maximum acceptable uncertainty should be stated in terms of a ratio  $\sigma(f)/\mu(f)$ , where  $\sigma(f)$  is the standard deviation (positive square root of the variance) and  $\mu(f)$  is the mean value of the power spectral density functions for the vibration environment covered by the specification. Note that the standard deviation as well as the mean value for the spectra is a function of frequency. Mean square values in narrow frequency intervals may be used instead of power spectra if desired.

# Step 5: Design the experiments needed to establish the environment. Assuming the spacecraft of interest has already been built, establish the number of flights, number of sample records per flight, and length of sample records required to define the environment within the maximum allowable uncertainty established in Step 4. If the spacecraft of interest has not been built, establish the type and amount of data needed from similar flight vehicles to permit a prediction of the environment within the maximum allowable uncertainty established in Step 4.

Step 6: Measure and/or predict the environment.

Assuming the spacecraft of interest has already been built, gather the necessary sample records established in Step 5 by appropriate flight tests, and reduce the sampled data. If the spacecraft of interest has not been built, gather the necessary data from measurements on similar flight vehicles or from theoretical considerations.

Step 7: Establish zones by pooling the detailed data into appropriate "equivalent groups."

Establish the minimum number of specification levels (zones) which may be employed to cover the entire vibration environment for the spacecraft in question. Pool all the data together which is appropriate for each specification level.

Step 8: Determine if the maximum uncertainty established in Step 4 has been met.

If not, determine the uncertainty associated with the data and revise the over-all uncertainty estimate.

Step 9: Establish test levels and test durations.

The general philosophy here should be that of simulating the actual environment. Test levels should not be increased above the actual environment except as needed to conservatively simulate the environment with an acceptable degree of uncertainty. The length of the test should be at least as long as the duration of the significant vibration to be expected in service. Test durations longer than the vibration exposure time in service should be based upon specific reliability considerations.

# Step 10: Perform the laboratory vibration test.

In line with the philosophy of simulating the actual environment, a random vibration test should be used to simulate random portions of the environment, and a sinusoidal vibration test should be used to simulate sinusoidal portions of the environment. Random-sine equivalences should not be used.

#### 5. DISCUSSION OF SUGGESTED APPROACH

## 5.1 SELECTION OF ASSEMBLY LEVEL FOR TESTS (STEP 1)

For purposes of final qualification and for acceptance testing, it is desirable that tests be performed on the most complete assembly feasible. For example, it is more desirable to qualify an equipment package by testing the entire package as a single unit than by testing each part and structure of the package separately. A single over-all test will clearly be a more accurate and dependable measure of equipment performance than a collection of parts tests. Hence, for the case of spacecraft applications, the most desirable level of testing for final qualification purposes would be to test the entire assembled spacecraft as a unit. If all dynamic inputs (including appropriate acoustical excitations) were properly simulated, then the vibration environment for all structures and parts would be accurately induced. Of course, because of the size of modern spacecraft, the required vibration and acoustical simulation facilities can become quite expensive.

In actual practice, tests at the equipment or individual part level are still required even if complete assembly tests are to be performed. This is true because the reliable performance of individual equipment packages and parts, which may be produced by many different manufacturers, must be verified by testing before they can be procured and installed into the complete spacecraft. Hence, the manufacturer of transistors requires a specification which is applicable to the vibration environment which his transistors will be expected to endure. The transistors may be indirectly tested at a later time as part of an equipment package or complete assembly test. In most cases, these later indirect tests will produce more accurate vibration inputs to the transistors. However, the test specification for individual transistors is still required to form a criterion for the design and final qualification of the transistors.

# 5. 2 VIBRATION LIFE HISTORY TO BE COVERED BY THE SPECIFICATION (STEP 2)

This second noted step might appear to be obvious. Nevertheless, it is often hastily considered or even completely ignored in actual practice. There is a natural tendency to emphasize the flight environment when developing a vibration test specification. However, the combination of factory handling, shipment, storage, and field installation could feasibly result in more damage than the actual flight environment. This is particularly true for the case of spacecraft where the flight environment is, relatively speaking, short in duration and usually not very severe.

All discussions in this document assume that the principal source of damaging vibration is the flight environment. However, it should be remembered that the vibration environment associated with factory handling, transportation, storage, and field installation could be important, and should be investigated. The vibration occurring in transportation from one point to another should be of particular concern because of the relatively long time intervals involved in transportation. Collections of vibration data for various modes of transportation are available from Reterences 44 and 45, which in turn include a number of additional pertinent references.

#### 5.3 PURPOSE OF THE SPECIFICATION (STEP 3)

Although an ideal vibration test specification should satisfy all purposes, practical circumstances often make it desirable to use slightly modified specifications for different applications. For example, if a test is being derived to evaluate the integrity of a component design, the risk of undertesting which one is prepared to accept may be somewhat less than for, say, a qualification test conducted to simply demonstrate proper component performance in a vibration environment. Hence, the vibration levels used for a design evaluation test may be somewhat greater than the test levels used for a qualification test. Furthermore, the test duration may also be greater to facilitate certain desired reliability conclusions. There is no reason, however, why the general nature of the test specification for these two applications should differ in any way other than the specified level and duration of vibration.

The situation is somewhat different for the case of design informa-The primary purpose of a design information tion and acceptance tests. test is to obtain specific engineering information concerning the dynamic characteristics of the component being tested. The primary purpose of an acceptance test is to detect poor workmanship. The successful performance of the component in a simulated service environment is of only secondary For these reasons, sinusoidal vibration testing is interest in either case. usually employed for design information and factory acceptance tests, even when the component of interest will be exposed to a basically random vibra-This is done because sinusoidal tion environment in actual service. excitations more readily permit the isolation and study of specific dynamics and/or workmanship problems. For similar reasons, sinusoidal vibration testing is sometimes used for design evaluation tests as well. In this case, however, sinusoidal testing should be employed only to support random vibration tests (assuming the environment is random).

The specification of exact frequency ranges, scan rates, and vibration levels for a design information test is usually not feasible, since such tests are principally of a research nature. Past experience indicates that formal test specifications for design information tests are not practical. Furthermore, the proper specification of acceptance tests is heavily dependent upon the specific manufacturing techniques which are used, and the type of workmanship errors which are expected. Hence, all further discussions in this document will apply to test specifications for design evaluation and qualification tests only.

#### 5.4 OVER-ALL UNCERTAINTY ESTIMATE (STEP 4)

No vibration test specification can be derived which will perfectly simulate a flight vibration environment of interest. There will always be some uncertainty as to how well the specified vibration test represents the details of the actual flight environment. Because of this uncertainty, the specified levels for the vibration test must always be higher than the estimated environmental levels to assure (with reasonable probability) that the vibration test levels are at least as severe as the actual vibration environment. The greater the uncertainty, the greater must be the specified vibration test levels to assure that the specification is adequately severe. Of course, increasing the specification levels to reduce the risk of undertesting will clearly increase the risk of overtesting (testing at levels which exceed the actual environment). The risk of overtesting for any given risk of undertesting may be reduced only by reducing the uncertainties associated with the derivation and implementation of the vibration test specification.

The over-all uncertainty for specification testing is a function of the separate variances associated with the principal steps required to write and implement the specification. In general, the principal steps involving pertinent uncertainties may be summarized as follows.

- a. The prediction of the vibration environment at structural locations where measurements are not obtained.
- b. The prediction of the vibration environment for future flights.
- c. The prediction of the vibration environment for spacecraft other than the spacecraft for which measured data is available.
- d. The use of stationary vibration data analysis techniques to analyze nonstationary vibration data.
- e. The use of a stationary vibration test to simulate a nonstationary vibration environment.

- f. Failure to simulate mechanical impedance characteristics (loading effects) in the vibration test.
- g. The use of specific test items to simulate the fragility of all production items of that design.
- h. The reproduction of the specified environment in the laboratory.

Note that this list of uncertainties assumes that the testing philosophy is one of simulating the actual environment. If an accelerated testing approach were planned, the above list of uncertainties would be substantially longer.

The above areas of uncertainty may be considered a linear combination of statistically independent sources of error. Hence, the total variance associated with the specification test will be equal to the sum of the individual variances for each of the above areas. The variance for each of these areas, and the parameters which control that variance, will now be discussed.

#### 5.4.1 Predictions for Structural Locations not Measured

The greatest uncertainty involved in the prediction of a space-craft vibration environment is that variability associated with spatial sampling considerations. In most cases, the practical limitations on the number of measurements which can be obtained prevents the measurement of the vibration response at each and every point of interest on the structure of the spacecraft. Hence, the vibration response at some points must be estimated based on measured data at other points.

The uncertainty introduced by these spatial sampling considerations is clearly a function of the zoning technique used to derive the specification. For example, if the zoning procedure separates basic frame structures from panel sections, the range of local vibration levels will not be as great as for the case where frames and panels are grouped together. The variance term in question can be reduced by increasing the number of zones (specified test levels) used to cover the spacecraft environment. Of course, this requires an increase in the number of points where measured or predicted data is available if the variance within each zone is to be properly defined. However, the variance can also be reduced by increasing the efficiency of the zoning procedure without increasing the number of zones or the required data. Any observed similarities in the vibration levels for specific types of structural design or construction should be exploited by making such structure a single zone.

For those cases where the vibration test specification is to apply to an entire spacecraft, the vibration environment of interest will be the motion at the interface where the spacecraft attaches to the launch vehicle. This motion plus appropriate acoustic excitation can be used to simulate the entire dynamic environment. In such cases, it is clearly desirable to develop only one vibration test level for the motion at the interface. Here, the data to be pooled together would be the vibration motion measured at all of the various attach points. It is hoped in such cases that the variance associated with the measured data will be relatively small. However, if the variance is not small, it must be carried along as an uncertainty in the resulting average levels. It is not practical in such cases to employ two or more vibration test specifications which are applicable to different attach points for the spacecraft.

# 5. 4. 2 Predictions for Future Flights

The vibration environment associated with a spacecraft launch is due principally to excitation forces which are stochastic in nature. Hence, the vibration response recorded on any given launch represents a unique set of circumstances which are never likely to be repeated. The following question then arises. How does the vibration environment measured on any given flight compare to the vibration environment to be expected on future flights? In other words, how much more severe might the actual environment be relative to the data measured on any one given flight.

For the case of data which is relatively stationary in time, the statistical uncertainty associated with measurements can be predicted from theoretical considerations. For example, given a specific measurement such as a power spectrum, a theoretical determination for the variance of the measurement may be obtained with a knowledge of the frequency bandwidth characteristics and sample record length in question. This variance may be reduced by increasing the length of sample records, as discussed in great detail in Reference 2 (Section 7), and Reference 3 (Section 1). However, spacecraft vibration environments are primarily nonstationary in nature. For this case, the theoretical development of variance expressions is not so straightforward. At the present time, the only way to obtain a good estimate for this flight-to-flight variance is to obtain data at the same structural locations on repeated flights and compute the variance.

# 5.4.3 Predictions for Vehicles not Measured

It is often required that vibration tests be specified and performed on components, or perhaps the entire spacecraft, before launch data from that particular spacecraft is available. Hence, it is then necessary to base specifications on vibration data obtained from some previous spacecraft and/or launch vehicles.

The prediction of vibration levels for one spacecraft based upon data measured from a different spacecraft will clearly involve an uncertainty which is a function of the magnitude of the difference between the two spacecraft and their launch vehicles, and the extrapolation procedures employed for the prediction (see Section 2.2 for a review of currently used prediction procedures, and Section 3.2 for criticisms of those procedures). The specific magnitude of this uncertainty is difficult to establish in any meaningful quantitative terms. However, it is clear that this uncertainty will be minimized by using data from a spacecraft whose construction and launch conditions are as similar as possible to the spacecraft of interest.

# 5.4.4 Stationarity Assumption for Data Analysis and Vibration Testing

As discussed in Sections 2.1.2 and 3.1, random vibration data analysis techniques are usually based upon time averaging procedures which inherently assume that the data in question is stationary in time. However, the vibration environment for spacecraft is, generally speaking, non-stationary in time. Hence, there is an area of uncertainty posed by the use of conventional stationary data analysis techniques to analyze non-stationary spacecraft vibration environments. The same problem arises when stationary vibration tests are used to simulate nonstationary spacecraft vibration environments, as discussed in Section 3.4.6. These problems are currently being studied and have not yet been fully resolved. However, preliminary results from Reference 38 indicate the statistical errors introduced by these factors are not severe, at least for the case of larger spacecraft where the launch phase acceleration is relatively low, assuming the data is properly interpreted.

#### 5. 4. 5 Mechanical Impedance Considerations

As discussed in Section 3.4.4, vibration test specifications for spacecraft components are sometimes based upon measurements on the unloaded supporting structure. As long as the item to be tested does not significantly load the supporting structure, the use of unloaded structural response data for specification writing purposes is acceptable. However, if loading effects are significant, a specification based on unloaded structural response data will produce an overly severe vibration test. This problem is clearly most severe for the testing of rather heavy components which are mounted in the spacecraft on relatively light supporting structures.

There are five possible approaches to the mechanical impedance problem. The first approach is to simply ignore the problem and accept the possibility of severe overtesting as an added safety factor in the design of the spacecraft components. For this case, no uncertainties would be considered in designing the test specification.

The second approach is to analytically consider the effects of impedance on the motion response for the loaded and unloaded supporting

structure, and to include some correction of these effects when establishing the vibration test levels to be specified. Past experience indicates that the best approach here is to use good engineering judgment when performing the vibration test. For example, if loading effects are significant in the actual service installation, one would not expect to see large motional inputs at those frequencies where a mounted component displays a resonance. The uncertainty associated with this approach can be assessed only on a basis of engineering judgment and past experience. Further technical discussions of these matters are available from References 40 and 41.

The third approach is to measure the actual impedance of the supporting structure for each component to be tested, and then simulate this impedance in the vibration testing machine. This would require advanced vibration testing machine circuitry to permit the simulation of an impedance for every supporting structure of interest. Such capabilities are not available at the present time and are not expected in the near future. It should be noted that the possibility of including provisions for a relatively crude simulation of mechanical impedance in vibration testing machines has been considered. However, this approach is of only limited value since slight differences in the simulated impedance characteristics for a supporting structure can have a significant effect on the response characteristics of a mounted component. It is very desirable that any impedance simulation be relatively accurate and, hence, customized for every component to be tested. If a relatively crude simulation of mechanical impedance is to be used, the natural impedance characteristics of the unequalized vibration testing machine may be as suitable as any.

The fourth approach is to test the components along with the basic structure to which they are attached in service. This in effect means increasing the assembly level for the test as discussed in Section 5.1.

The fifth approach is to obtain measurements of the vibration response in actual service with all components installed. Mechanical impedance considerations will be accounted for in the motion response measurements. Of course, this would require tests on the actual vehicle and components of interest, which is usually not feasible.

# 5.4.6 Fragility Level of the Test Item

The fragility level of a component is defined as that vibration level which will cause failure. The fragility level is generally a function of frequency as well as exposure time. Hence, the fragility level for a component is usually displayed in terms of a three-dimensional plot of fragility versus frequency and time. This plot is referred to as a fragility surface. General techniques for establishing fragility surfaces for flight vehicle components are presented in References 46 and 47.

The problem here is that the fragility surface for a component will vary somewhat from one item to another due to slight manufacturing and materials differences. Hence, one sample of a given component might fail a specified vibration test while another would not, or vice versa. In many cases, this variation is undoubtedly negligible. However, experience indicates that the fragility level at critical frequencies of some active components may vary from one sample to another by factors of over two. This is particularly true of electronic and/or electro-mechanical components which may fail due to vibration induced electrical noise.

Unfortunately, there is very little quantitative data available on the variance of fragility surfaces for components. The reason is obviously the large amount of testing which must be performed to obtain this type of information. This general area is in need of additional study.

# 5. 4. 7 Reproduction of Specified Vibration

The accuracy with which the vibration test specification is reproduced in the laboratory is a function primarily of the equalization of the vibration testing machine. For the case of random vibration tests (which are of particular interest here), equalization is accomplished by shaping

the power spectrum of the armature signal using a collection of contiguous narrow bandpass filters, as previously mentioned in Section 2.5. However, these bandpass filters are, generally speaking, relatively broad on modern vibration testing machines. Relatively broad here means that the equalizing filters may sometimes be wider than sharp peaks or notches observed in the armature motion due to the resonant response characteristics of the component being tested. Hence, a perfect reproduction of the specified vibration motion can never be achieved. There will always be some deviation in the actual motion produced in the laboratory from the specified motion. The variance for this deviation from the specified motion might be quite large for the case of extremely complicated components. For example, an uncertainty of 50% or more is relatively common. A more precise definition for the uncertainty associated with vibration tests must be established in terms of the specific vibration testing machines employed and the specific component to be tested.

#### 5.5 DESIGN OF EXPERIMENTS (STEP 5)

This important step is present in all previous procedures for creating vibration test specifications, although it is often considered hastily. This is due in part to the fact that properly designed flight test experiments can rarely be implemented for spacecraft applications to the extent desired. The availability of telemetry channels and the high cost of repeated launches impose severe restrictions on the flexibility needed for the design of a statistically meaningful flight test program. These facts coupled with the nonstationary nature of spacecraft vibration data tend to limit the application of detailed statistical design techniques, as discussed in Section 2.1. The design of a spacecraft flight test experiment usually reduces to the acquisition of as much data as permitted by practical considerations, which is rarely enough data to satisfy the desired requirements.

There is, however, at least one factor in the experimental design which can be controlled. This involves the selection and location

of transducers. Since the transducers often constitute the only flexible factor in the experiment, great care should be exercised in their selection and location. A few important guidelines are listed below.

- (a) The frequency response range and physical size of the transducer should be emphasized over nominal accuracy figures for linearity, sensitivity, etc. The relatively large statistical uncertainties associated with the analysis and final use of the data will generally overshadow any reasonable accuracy characteristics for commercial transducers. For example, one should never sacrifice data in a frequency range of interest to obtain an improvement in sensitivity accuracy from, say, 5% to 1%.
- (b) Transducers should always be located on principal structures and as near as possible to the attachment points for those components for which specifications are to be derived.
- (c) The mounting brackets for the transducers should be carefully designed and, if necessary, tested to assure that the frequency response function through the bracket (with the transducer attached) is near unity for all frequencies of interest.
- (d) The weight of the transducers and their mounting brackets should be as small as feasible to minimize their loading effects on the structure to which they are attached. The possibility of significant loading effects will be further reduced by assuring that the transducer brackets are attached directly to the principal structures such as frame sections, and not to panel sections or weak intercostal structures.

# 5.6 DETERMINATION OF THE ENVIRONMENT (STEP 6)

For the case where the spacecraft of interest has already been built, the flight vibration environment should be established by direct measurements at the desired structural locations during launch (and re-entry if applicable). The ultimate goal, of course, would be to obtain direct measurements of the vibration at or near all points of attachment for the components to be tested.

The measurements at various points should be analyzed in terms of a frequency composition (such as a power spectrum). Of particular interest is spectral data for critical nonstationary phases during the launch such as lift-off, transonic flight, and maximum dynamic pressure. One approach is to measure a continuous time varying spectrum by using a parallel filter type instrument with a relatively short averaging time, as discussed in References 7, 8, 9, and 10. The peak spectrum which occurs during each flight phase of interest may then be extracted from the continuous plot. A second approach is to form continuous loops from short sections of the sample record covering those critical phases of interest, and then to measure a spectrum for each loop using conventional spectral analysis techniques as discussed in Reference 6. Either of the above two measurement and spectral analysis procedures are acceptable in lieu of improved methods for analyzing and interpreting nonstationary data, which are currently being studied in Reference 38.

For the case where the spacecraft of interest has not been built, the environment should be predicted by extrapolation of data from previous launches of other spacecraft. Clearly, to minimize extrapolation errors, the data used for the predictions should be from a spacecraft whose construction and launch conditions are as similar as possible to those for the spacecraft of interest. The vibration predictions should be made, at least as a first step, for individual point locations. That is, the general prediction techniques discussed in Section 2.2.1 should not be used unless no other data is available. Otherwise, any of the currently available custom prediction techniques reviewed in Section 2.2.2 may be employed in lieu of improved procedures.

#### 5.7 GROUPING OF DATA INTO ZONES (STEP 7)

Having measured or predicted the vibration response at various structural locations and perhaps various times as well, it is now necessary to pool the resulting data into groups where each group will be associated with a particular specification test. The goal is to create as few groups as possible while still maintaining an acceptable variance for the data within each group.

This step is the most significant of all in controlling the over-all variance of the final environmental estimates, because the manner in which the spacecraft structure is zoned will greatly influence the spacial distribution of vibration levels within each zone, as discussed in Section 5.4.1. Various concepts of data grouping are discussed in Sections 2.3 and 3.3. The important requirement here is to emphasize similarities in the data as a basis for data grouping rather than pure regional zoning considerations. The variance introduced by the grouping procedure can be calculated directly from the data within each group.

After pooling the basic data into appropriate groups, an average spectrum should be calculated for each group. These average spectra will form the basis for establishing specification test levels.

#### 5.8 REDETERMINATION OF OVER-ALL UNCERTAINTY (STEP 8)

With the actual collection and analysis of the desired data complete, a second look should be taken at the original uncertainty estimate (Step 4). Unquestionably, there will have been many problems and practical considerations which prevented the acquisition of all the data needed to comply with the original estimate. Hence, the original uncertainty estimate should be revised if necessary to reflect the best estimate for a total variance available after the data acquisition and analysis is complete. Note that the resulting variance estimate for each specification will be a function of frequency. The most significant contribution to the over-all variance will generally come in Step 7.

# 5.9 ESTABLISHMENT OF TEST LEVELS AND DURATIONS (STEP 9)

#### 5.9.1 Establishment of Test Levels

The average spectrum for an equivalent stationary vibration environment to be covered by each specification has been estimated in Step 7, and the associated uncertainty is determined in Step 8. The only remaining information required to establish a proper level for testing is a statement of the risk of undertesting which one is prepared to accept and an estimate of the probability density function (sampling distribution) for the spectra of vibration levels within each zone. With this information, a "raw spectrum" for the test level may be established as follows.

$$T(f) = \mu(f) + k\sigma(f)$$

Here,  $\mu(f)$  is the average and  $\sigma(f)$  is the standard deviation for the spectral levels in the zone of interest, and k is a constant which is dependent upon the assumed sampling distribution and desired percentile level for the test. A "smoothed spectrum" for the test level should then be established by enveloping the raw spectrum with straight line segments when displayed on a log-log scale. A sufficient number of straight line segments should be used to permit a reasonable fit to the predominant peaks and valleys of the raw spectrum, as illustrated in Figure 3.

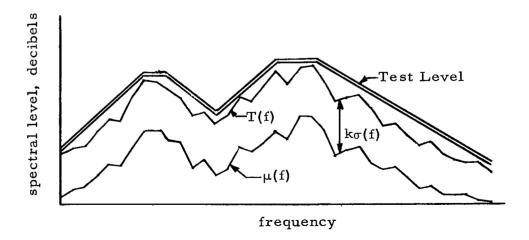


Figure 3. Raw and Smoothed Spectra for Test Levels

It should be mentioned that the bandwidth of spectral data used to establish the test levels can influence the results. Specifically, if the filter bandwidth used for the original spectral analysis is substantially wider than the bandwidth of spectral peaks in the data, the tendency is to reduce the variance of spectral measurements within a zone. This is caused by the additional averaging introduced by frequency smoothing. Hence, the resulting specified test levels will also be reduced, making the test less conservative. This problem will be minimized if the filter bandwidth used for spectral measurements in data reduction is less than about 5% of center frequency (B < 0.05f). However, even if wider filters are used for data analysis, the frequency averaging effect can be accounted for by adding a simple correction factor. For example, Reference 16 indicates that a factor of about 3 db will account for differences between a properly resolved power spectrum and one which is computed using one-third octave bandwidth filters (B = 0.22f) for the case of missile launch vibration data.

The principal problem in arriving at the raw spectrum for the test level is the determination of an appropriate value for k. This in turn requires an estimate for the sampling distribution of the spectral data within a given zone. The theoretical determination of this sampling distribution is not feasible because of the complexity of the factors which contribute to the random variable of interest. For those cases where the uncertainty is less than about 33% (the standard deviation is less than about one-third the estimated average spectrum level), a normal approximation for the sampling distribution is probably acceptable. However, the uncertainty for the spectral values will usually be much larger than this amount in practice. Since a spectrum can never take on negative values, a normal distribution is clearly not an acceptable approximation when the uncertainty is large. Some skewed type of distribution function is needed.

As discussed in Section 2.4.1, the log-normal distribution is often assumed for the sampling distribution of spectral levels when considered in terms of mean square values in narrow frequency intervals. When feasible, a better approach would be to estimate a sampling distribution function (or a desired percentile level) by empirical studies of the specific data, as was done in Reference 29. However, in the absence of specific data, the log-normal assumption is probably as good as any, although experience indicates it tends to produce conservative percentile level estimates. Another approach to this problem is to simply ignore the detailed sampling distribution and arbitrarily use a value of k between 2 and 3. This corresponds to a 97.7 to 99.9 percentile level for a normal distribution and a somewhat lower percentile level for most skewed distributions.

# 5.9.2 Establishment of Test Durations

With the specification test levels established, it is now necessary to determine a test duration. The logical procedure for selecting a test duration is to use the vibration exposure time to be expected in actual service. However, the exposure time may not be clearly defined for the case of spacecraft. For example, if the exposure time is considered to be the entire launch time, and this is used to establish the test duration, the resulting test will clearly be too severe since the test levels are based only upon the maximum vibration levels which occur during launch. On the other hand, if the test duration is based upon the time duration for the maximum vibration levels alone, the resulting test may not be sufficiently severe since the vibration at those times when levels are not a maximum would not be accounted for.

There are technical ways to arrive at a reasonable test duration. One way is to equate the duration of the time varying vibration environment during launch to the duration of a stationary vibration environment which would produce an equivalent amount of fatigue damage. Another

way is to equate the duration of the nonstationary environment to the duration of a stationary environment which would produce the same number of extreme peaks. The relationships needed to establish equivalence based upon either a fatigue damage criterion or a peak criterion are presented in Reference 48, Sections 7 and 8. Although the above suggestions form helpful guides for establishing test durations, engineering judgment and past experience are still the most valuable factors needed to arrive at proper conclusions.

# 5.9.3 Extended Test Durations Based on Reliability Considerations

The approach to selecting a test duration suggested in the preceding section does not really take reliability considerations into account. More specifically, the ultimate goal of any testing program should be to permit the following question to be answered. If a component performs properly during a given test, what is the probability that this component will perform properly in actual service?

The answer to the above question is a straightforward engineering reliability problem. Generally speaking, proper reliability conclusions based on test results require (a) testing of several different samples of each test item and/or, (b) testing to failure (destructive testing). For the special case of vibration tests for spacecraft components, either repeated tests or destructive tests may be difficult to arrange due to the high cost of sample test items. However, if one is prepared to make certain critical assumptions concerning the nature of expected failures, some reliability information can be extracted from single tests on single samples of the test items where failure does not occur.

Given a component which is to be tested, if it is assumed that failures will occur randomly (an exponential failure rate applies), then the reliability of that component, as defined by the mean-time-between-failure (MTBF), may be described by the following equation.

$$P_0 = e^{-t/t_0}$$

Here,  $P_0$  is the probability that no failure will occur in t seconds, and  $t_0$  is the MTBF in seconds.

The above relationship may be used to establish a hypothesis test as follows. Let it be hypothesized that the MTBF for a particular component is  $t_0$  or less when that component is exposed to its expected service life vibration environment. Now, if the component is vibration tested for a time duration such that  $t >> t_0$ , then the probability of no failures, as given by  $P_0$ , will be small. Hence, if no failures occur, the above hypothesis would be rejected at the  $P_0$  level of significance since the occurrence of no failures is highly unlikely if the MTBF were actually  $t_0$  or less. In other words, the occurrence of no failures after  $t >> t_0$  seconds of testing means that the MTBF is probably greater than  $t_0$ .

For example, if it is desired to establish that the MTBF for a component is greater than  $t_0$  at the  $P_0$  = 0.05 level of significance, the required test duration without a failure is

$$t = -t_0 \ln P_0 = 3.0 t_0$$

That is, the test duration would have to be at least three times longer than the minimum desired MTBF for the component.

It is clear that the desired MTBF for the component would be very much longer than the expected vibration exposure times in actual service. Hence, test durations would also be very much longer than the expected exposure times. This poses a serious limitation on the application of these ideas to any test items where wear-out or fatigue failures might occur. Remembering the original assumption that failures occur randomly, the procedure does not allow for the possibility of fatigue or

other such non-random failures. Hence, extending the test duration might cause such failures when in fact they would not occur in actual service.

In conclusion, the proper way to introduce reliability aspects into a testing program is to perform either repeated tests on many different samples and/or destructive tests. Further information on these matters is available from Reference 49.

## 5.10 PERFORMANCE OF VIBRATION TEST (STEP 10)

Given a specified vibration test in terms of spectra for stationary random and/or periodic vibration environments, the test may be performed using currently available vibration testing machines as discussed in Section 2.5. In order to eliminate the uncertainties posed by random-sine equivalences, random vibration should be used to simulate the random portions of the environment and sinusoidal vibration should be used to simulate the sinusoidal portions of the environment. The random and sinusoidal portions of the test should be performed simultaneously where possible, assuming they occur simultaneously in service.

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Reference 13 presents the general concepts of Fourier Spectra and Shock Spectra, and discusses their applications to the analysis and interpretation of shock movements.

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Reference 14 develops a technique for predicting flight vehicle vibration levels based upon empirical correlations between the external sound pressure field and structural vibration measurements from the B-58 aircraft.

15. Eldred, K., Roberts, W.M., and R. White, "Structural Vibrations in Space Vehicles," WADD TR-61-62, Wright Air Development Division, ARDC, USAF, Wright-Patterson AFB, Ohio. 1961.

Reference 15 presents broad discussions and considerable background material on the prediction of vibration environments in space vehicles and the creation of test specifications from predicted environments.

Brust, J. M., and H. Himelblau, "Comparison of Predicted and Measured Vibration Environments for Skybolt Guidance Equipment," Shock, Vibration and Associated Environments Bulletin No. 33, Part III, pp. 231-280, Dept. of Defense, Washington, D. C. March 1964.

Reference 16 outlines a prediction procedure based upon empirical correlations similar to Reference 14. Specific results are presented and compared to the results obtained using other prediction techniques and the results of actual measurements.

- 17. Powell, A., "On the Response of Structures to Random Pressures and to Jet Noise in Particular," Chapter 8, Random Vibration, edited by S.H. Crandall, John Wiley and Sons, Inc., New York. 1959.
- 18. Franken, P.A. et al, "Methods of Space Vehicle Noise Prediction," WADC 58-343, Wright Air Development Division, ARDC, USAF, Wright-Patterson AFB, Ohio. Vol. I, November 1958, Vol. II, December 1960. (AD 205 776).
- 19. Barnoski, R.L., "Response of Elastic Structures to Deterministic and Random Excitation," AFFDL TR-64-199, Research and Technology Division, AFSC, USAF, Wright-Patterson AFB, Ohio. December 1964.

References 17 through 19 present basic material on the classical approach to calculating the vibration response of flight vehicle structures to random excitation. Reference 19 in particular includes many numerical examples.

20. Butler, T.G., "The Value of Limited Random Vibration Flight Data," Paper No. 925B, SAE National Aeronautic and Space Engineering and Manufacturing Meeting, Los Angeles, California. October 1964.

Reference 20 discusses the type of data required to establish input-output relationships for structures, as required for classical prediction techniques. Vector notation is used in derivations. The problems posed by insufficient flight data are considered.

21. Enochson, L.D., "Frequency Response Functions and Coherence Functions for Multiple Input Linear Systems," NASA CR-32, National Aeronautics and Space Administration, Washington, D.C. April 1964.

> Reference 21 presents procedures for measuring inputoutput relationships for structures. Detailed derivations of basic results and numerical illustrations are included.

22. Lyon, R. H., "An Energy Method for Prediction of Noise and Vibration Transmission," Shock, Vibration and Associated Environments Bulletin No. 33, Part II, pp. 13-25, Dept. of Defense, Washington, D. C. February 1964.

Reference 21 reviews a general method for estimating the vibrational energy of structures subjected to either acoustic excitations or mechanical excitations from other attached structures. The method yields average vibration response predictions with a defined statistical uncertainty.

- 23. Baker, W. E., et al, "Colloquium on Use of Models and Scaling in Shock and Vibration," American Society of Mechanical Engineers, New York. 1963.
- 24. Greenspon, J.E., "Modelling of Spacecraft Under Random Loading," NASA CR-132, National Aeronautics and Space Administration, Washington, D.C. November 1964.

References 23 and 24 discuss the use of scale models for struc tural analysis and vibration prediction studies.
Reference 23 is a collection of eight papers on the subject.
Reference 24 deals specifically with scaling laws for structures subjected to random excitation.

25. Barnoski, R.L., and C.R. Freberg, "Passive Element Analog Circuits for Three-Dimensional Elasticity," Paper No. 65-AV-4, ASME Aviation and Space Conference, Los Angeles, California. March 1965.

Reference 25 presents a procedure for deriving passive element analogs for three dimension structures. Such analogs are a basis for passive analog models which can be used for the study of flight vehicle structural vibration.

- 26. Condos, F.M., and W.L. Butler, "A Critical Analysis of Vibration Prediction Techniques," Proc. Inst. of Environmental Sciences Annual Technical Meeting, pp. 321-326. 1963.
- 27. Bolt, Beranek and Newman, Inc., "Procedures Utilized in Developing All-Random Vibration Test Specifications for Titan III," BBN Report No. 1083 (Job No. 111222). 27 January 1964.

28. Barrett, R. E., "Techniques for Predicting Localized Vibration Environments of Rocket Vehicles," NASA TN D-1836, National Aeronautics and Space Administration, Washington, D. C. October 1963.

References 26 through 28 outline vibration prediction techniques using simple extrapolation formulae. Data measured in some previous vehicle is manipulated to apply to a new vehicle based upon excitation and structural mass differences. References 26 and 27 also outline procedures for arriving at test levels based upon predicted vibration environments.

29. Barret, R. E., "Statistical Techniques for Describing Localized Vibratory Environments of Rocket Vehicles," NASA TN D-2158, National Aeronautics and Space Administration, Washington, D. C. July 1964.

Reference 29 outlines a procedure for arriving at test levels for launch vehicle components based upon predicted vibration environments. The procedure is based upon an empirical relationship developed after detailed studies of large quantities of applicable data.

Miner, M.A., "Cumulative Damage in Fatigue," <u>Journal of Applied Mechanics</u>, Vol. 12, pp. 159-164. September 1945.

Reference 30 introduces the concept that fatigue failures are the result of an irreversable accumulation of damage. This concept in one form or another is widely used to predict fatigues in flight vehicle structures.

31. Miles, J.W., "On Structural Fatigue Under Random Loading,"
Journal Aero. Science, Vol. 21, pp. 753-762. November 1954.

Reference 31 develops an expression for the fatigue damage accumulated in structures subjected to random excitation, using the concepts of Reference 30. The expression developed is widely used as a basis for establishing an equivalence between random and sinusoidal vibration.

32. Spence, H.R., and H.N. Luhrs, "Peak Criterion in Random vs. Sine Vibration Testing," Journal of Acoustical Society of America, Vol. 33, No. 5, May 1961.

Reference 32 outlines a method for comparing the severity of random and sinusoidal vibrations which is not based directly on an equivalent fatigue damage criterion.

33. Booth, G., "Sweep Random Vibration," Proc. Inst. of Environmental Sciences, Annual Technical Meeting, Los Angeles, California. 1960.

Reference 33 suggests the use of a swept narrow frequency bandwidth random vibration test to simulate broadband random vibration environments.

- 34. Crede, C. E., Gertel, M., and R.D. Cavanaugh, "Establishing Vibration and Shock Tests for Airborne Electronic Equipment," WADC TR-54-272, Wright Air Development Center, ARDC, USAF, Wright-Patterson AFB, Ohio. 1954. (AD 45-696).
- 35. Gertel, M., "Establishing Vibration and Shock Test Procedures for Air-Borne Electronic Equipment," Barry Controls Interim Report to USAF, Contract AF33(038)-22704, Barry Controls, Inc., Watertown, Mass. 1955.

References 34 and 35 cover early studies of the general problem of deriving vibration test specifications for aircraft equipment. Emphasis is placed upon procedures where thousands of hours of service life are simulated by a few hours of testing.

- 36. Gertel, M., Specification of Laboratory Tests, "Chapter 24, Shock and Vibration Handbook, edited by C. M. Harris and C. E. Crede, McGraw-Hill Book Co., New York. 1961.
- 37. Gertel, M., "Derivation of Shock and Vibration Tests Based on Measured Environments," Shock, Vibration and Associated Environments Bulletin No. 31, Part II, pp. 25-33, Dept. of Defense, Washington, D.C. April 1963.

References 36 and 37 discuss the derivation of vibration test specifications based upon a simulation of the damaging effects of the environment. Cumulative fatigue is the damage criterion used.

38. Piersol, A.G., "Spectral Analysis of Nonstationary Spacecraft Vibration Data," submitted to Goddard Space Flight Center under Contract No. NAS 5-4590.

Reference 38 covers detailed experimental studies of the basic characteristics of nonstationary spacecraft vibration data. Improved techniques for analyzing nonstationary vibration data with conventional time averaging instruments are suggested.

39. Barnoski, R.L., "Response of Mechanical Systems to Nonstationary Random Excitation," submitted to Goddard Space Flight Center under Contract No. NAS 5-4590.

Reference 39 reviews theoretical and experimental studies of the response of linear systems to nonstationary random excitation. One and two degree-of-freedom systems are considered along with several different types of nonstationary input functions.

40. Pulgrano, L.J., "Impedance Considerations in Vibration Testing," Shock, Vibration and Associated Environments Bulletin No. 31, Part II, pp. 236-244, Dept. of Defense, Washington, D.C. March 1963.

Reference 40 presents a practical discussion of mechanical impedance and its influence on the selection of proper test levels. The paper illustrates for the case of a spacecraft how a failure to consider impedance factors can result in severe overtesting.

41. On, F.J., and R.O. Belsheim, "A Theoretical Basis for Mechanical Impedance Simulation in Shock and Vibration Testing of One-Dimensional Systems," NASA Technical Note D-1854, Goddard Space Flight Center, NASA, Greenbelt, Maryland. August 1963.

Reference 41 presents the theoretical background of mechanical impedance and summarizes analytical procedures for applying impedance concepts to vibration problems.

42. Blake, R. E., "A Method for Selecting Optimum Shock and Vibration Tests," Shock, Vibration and Associated Environments Bulletin No. 31, pp. 88-97, Part II, Dept. of Defense, Washington, D. C. 1963.

Reference 42 suggests the application of statistical decision theory to the derivation of shock and vibration tests. An excellent qualitative discussion of the uncertainties associated with the specification problem is presented.

43. Bangs, W.F., "Sinusoidal Vibration Testing of Nonlinear Spacecraft Structures," NASA Technical Note D-1763, Goddard Space Flight Center, NASA, Greenbelt, Maryland. 1963.

Reference 43 discusses the practical problems produced by non-linear response characteristics during sine wave vibration tests. Mechanical impedance considerations are noted.

- 44. Vigness, I., "Field Measurements, Specifications, and Testing," Chapter 8, Random Vibration, Vol. 2, edited by S.H. Crandall, MIT Press, Massachusetts Inst. of Technology, Cambridge, Mass. 1963.
- 45. Engelhardt, R. E., Mills, K. D., Schneider, K., and S. G. Guins, "Shock and Vibration in Road and Rail Vehicles," Chapter 45, Shock and Vibration Handbook, edited by C. M. Harris and C. E. Crede, McGraw-Hill Book Co., New York, 1961.

References 44 and 45 include considerable data on the shock and vibration environments for transportation by trucks, railroads, ships, and airplanes.

- 46. Brust, J.M., "Determination of Fragility to Meet Random and Sinusoidal Vibration Environments," Paper 430A, SAE National Aeronautic and Space Engineering and Manufacturing Meeting, Los Angeles, California. October 1961.
- 47. Moulding, E.L., "Derivation of Random Fragility from Sinusoidal Tests," Paper 752C, SAE National Aeronautic and Space Engineering and Manufacturing Meeting, Los Angeles, California. September 1963.

References 46 and 47 present detailed discussions of methods for establishing fragility levels for airborne equipment.

48. Bendat, J.S., "Probability Functions for Random Responses:
Prediction of Peaks, Fatigue Damage and Catastrophic Failure,"
NASA CR-33, NASA, Washington, D.C. April 1964. (N64-17990).

Reference 48 discusses the practical interpretations of probability density and distribution functions, and applications for structural vibration problems. The prediction of long term structural fatigue damage as well as short term catastrophic failures due to extreme peaks are covered.

49. RADC Reliability Notebook, RADC TR-58-111, Rev. 31, Rome Air Development Center, Griffiss AFB, Rome, New York. December 1961.

Reference 49 presents broad background material concerning reliability theory and its applications to practical engineering problems.